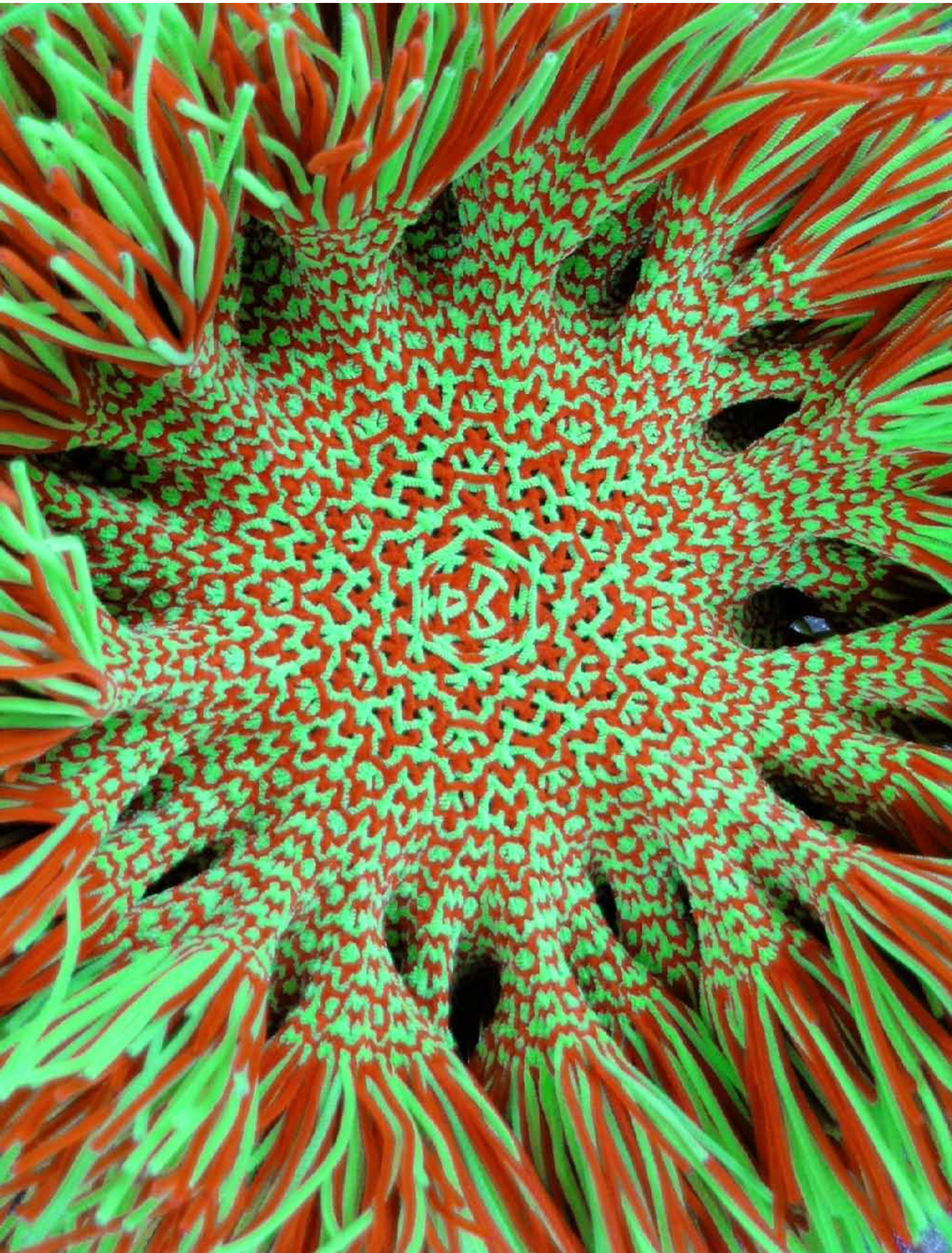
The National Museum of Mathematics *presents:*



COMPOUNDING VISIONS

The art of Trevor and Ryan Oakes

The National Museum of Mathematics
presents:

COMPOUNDING VISIONS

The art of Trevor and Ryan Oakes

Curated with texts by
Lawrence Weschler

Conjured into being
at the instigation of
Mark Mitton

11 East 26 Street
New York City

May 10 - July 21, 2014

Beauty, creativity, exploration…

The ability to enhance perceptions and change perspectives…
The simple elegance—or the delicious complexity—of pattern, texture, and color…

Art, or math?

There's an artistry to mathematics that captivates those who peer closely…
and mathematical interpretations of art that can illuminate and elucidate.

Welcome to *Composite*, the gallery at MoMath, where art and math intersect to provide a unique perspective: a chance to perceive the world from a different angle, and to come away with a new, often surprising, understanding.

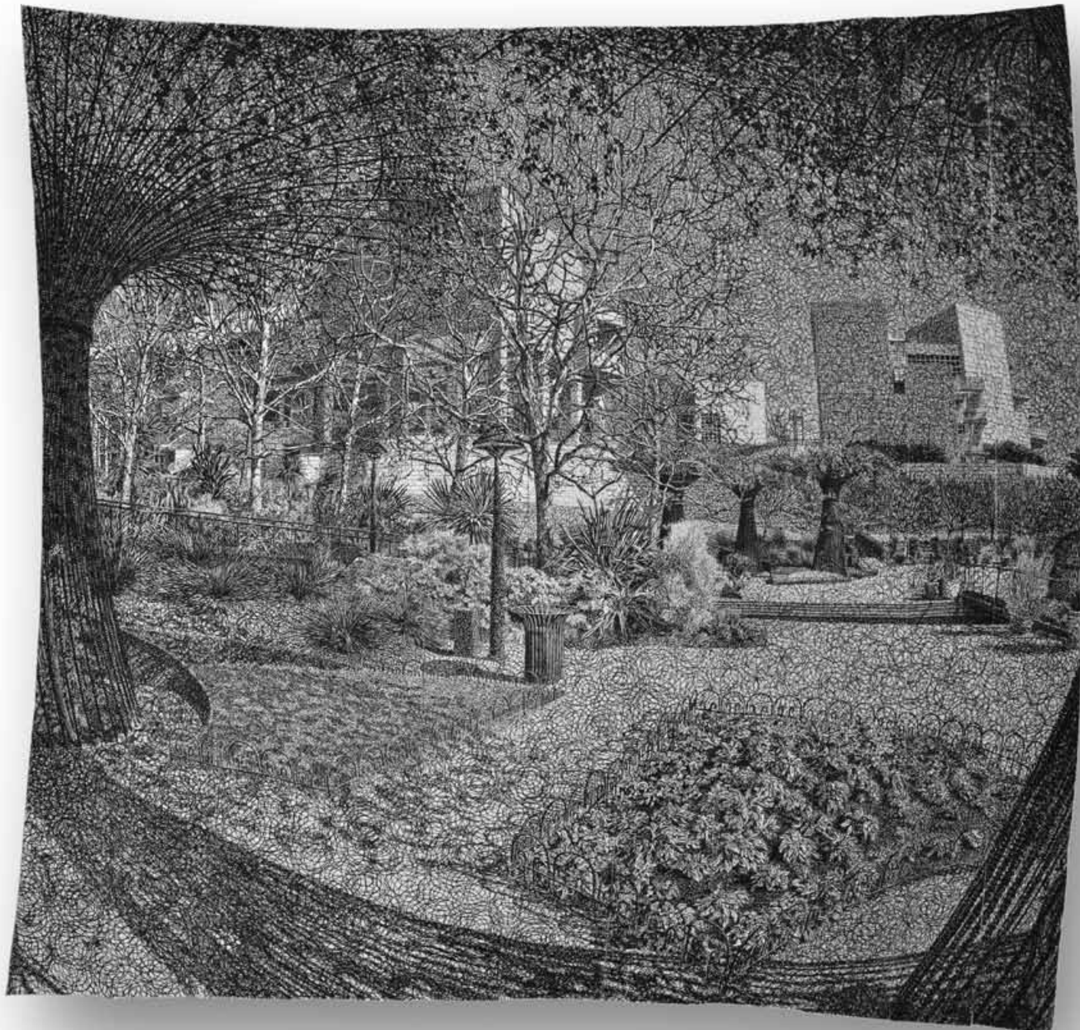
The National Museum of Mathematics is delighted to open *Composite* with the work of Trevor and Ryan Oakes. These young artists explore perspective and technique in a way that bridges two worlds, expressing both the mathematical nature of art and the artful nature of mathematics. We hope you will join us in enjoying, engaging, and discovering.

Cindy Lawrence

Cindy Lawrence, Co-Executive Director
National Museum of Mathematics

SPECIAL THANKS TO:

Suzanne Cruse
Halley K. Harrisburg
Chuck Hoberman
Joseph J. Kohn
Mark Mitton
Ryan Oakes
Trevor Oakes
Francesco Scattone
Lawrence Weschler



LAWRENCE WESCHLER

INTRODUCTION

The Twins and I met by way of the other twins. This was back in 2007, and they were only twenty-five at the time. We were all attending the opening at the Metropolitan Museum of an installation by the artist Tara Donovan, and the other twins—Margaret and Christine Wertheim, the one a physicist-manqué science writer and the other a poet/literary theorist, the two of them founders of the Institute for Figuring in Los Angeles, an entire enterprise given over to celebrating the aesthetic splendors of scientific thinking, which is to say all the beauty that is in truth and vice versa—anyway, the Wertheim Twins were telling me that the guys I really had to meet were these other twins, the Oakes boys.

Now, the Wertheims had in those days only just recently launched the hyperbolic crocheted coral reef (that marvelous interpenetration of non-Euclidean mathematics, environmentalist concern, feminist handicrafts, aesthetic marvel and political activism—granted, a whole other story, but one well worth Googling), and they were telling me that if I thought that that was something (which I fervently did), then I'd really love what Trevor and Ryan were up to, with pipe cleaners of all things! Which, indeed, within a few hours proved decidedly the case.

For the Oakes boys, too, were clearly forging their own way into the terrain between the aesthetic and the investigatory—which is to say the same terrain all artists used to inhabit before the relatively recent and entirely artificial rise of the divide between the arts and sciences. The sort of terrain that Leonardo and Michelangelo and Brunelleschi would have called their own (imagining no other). For these kids—and they were very young: bright and vivid and unabashedly open—were at the same time age-old wise. As became clear in talking to them, they had been involved since toddlerhood in a progressively ever-more-consuming dialog on the nature of perception (two boys talking hour after hour, month after month, year after year, about what it is like, precisely, to see with two eyes), so they'd already been at it for a long time. And though their path had been entirely "artistic"—they'd only just graduated from Cooper Union—it was clear that the sorts of issues they were exploring ramified in all kinds of other directions as well.

The artist Robert Irwin, whose thinking and career I'd explored years earlier in my own first book, *Seeing is Forgetting the Name of the Thing One Sees*, likes to say that it's not all that surprising that explorers at the edges of their respective disciplines (physicists, philosophers, artists, mathematicians, architects, biochemists, and the like) keep bumping up against each other, because they are all engaged in what he likes to call "the dialog of immanence." And from our first meetings it was clear to me that these kids were deeply engaged in that kind of liminal inquiry.

So, to make a long story short, I wrote about them and their adventures relatively early on in a piece for the Virginia Quarterly Review—the very piece that follows – one that I hope will give you a general background on where the boys are coming from, and a hint of where they were headed back then, in 2009. And I was hardly surprised, a few years after that, to learn that the National Museum of Mathematics had decided to devote the first show in their new art gallery space to the two of them. Despite the fact that the Twins had no particular mathematical training, it was obvious that they had deeply mathematical minds, which is to say sensibilities besotted with the splendors of pattern and order.

Still, little did I know what an adventure it would prove, when, through the ministrations of Cindy Lawrence, Glen Whitney and Tim Nissen of the Museum, and especially thanks to the interventions and introductions provided by their pal, that other sublime inquirer after immanence, the magician (!) Mark Mitton, the Twins actually began engaging with real-life mathematicians, folks like Joe Kohn and John Conway and Manjul Bhargava of Princeton, and Chaim Goodman-Straus of the University of Arkansas, and George Zweig (originally out of Los Alamos and MIT).

The results of those conversations are everywhere evident in the show that resulted and the catalog that follows, and our appreciation to all of those mentioned above is boundless. Likewise to Gerri Davis who designed the catalog—no mean feat, as you will come to understand.

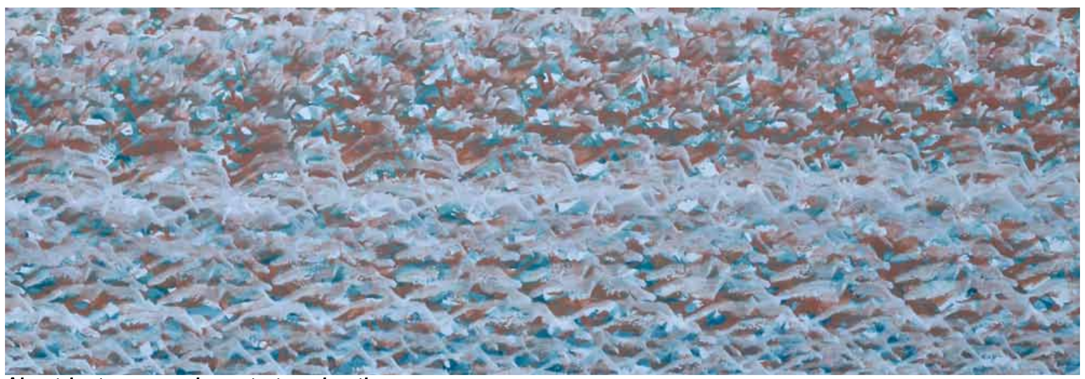
So, turn the page and meet Ryan and Trevor Oakes by way of that original VQR piece, after which you too will get a chance to see what it can be like when disciplines collide and conjoin and ramify away…what it is like, in short, to positively revel in *figuring stuff out*.

MATHEMATICAL IMPLICATIONS

So, as we say, the Oakes Twins were not themselves mathematicians,
nor had they ever had any particular mathematical training.
But their work kept tending into distinctly mathematical terrain,
and this show proposes to explore some of those incursions.
Thus, for example:

ALGORITHMIC WATERCOLOR PAINTINGS

Throughout their practice, the Twins keep reverting to *algorithmic procedures*, which is to say they set themselves a procedural rule which they then follow all the way through. Thus, for example, in the series of paintings represented by the two examples to your right, they started out by inventing a brush (a scrap of felt twisted and folded and lashed around the tip of a stick, below) that they saturated in blue paint. They then took a long vertical piece of rice paper, and starting at the bottom, rolled the blue brush horizontally along the paper, initially with the brush-stick itself held almost parallel to the surface. The next line up, they held the stick ever so slightly more vertically, again rolling it in a straight line from one side of the paper to the other. And again, and again, each time holding the brush-stick at a slightly higher angle and therefore traversing from one edge of the paper to the other ever more slowly, till, at the top of the sheet of rice paper, the brush was being twirled upon its tip at a 90-degree angle, perpendicular to the paper, making tiny strokes and tumbling along at an almost imperceptible pace. The result, curiously, read as a receding field of waves.



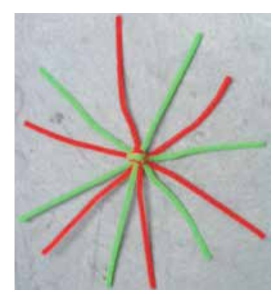
Algorithmic watercolor painting, detail

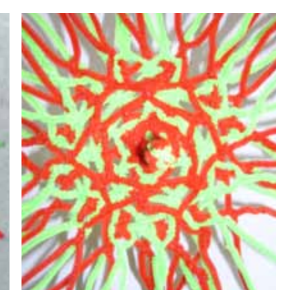
Handmade felt brushes developed for algorithmic watercolor paintings

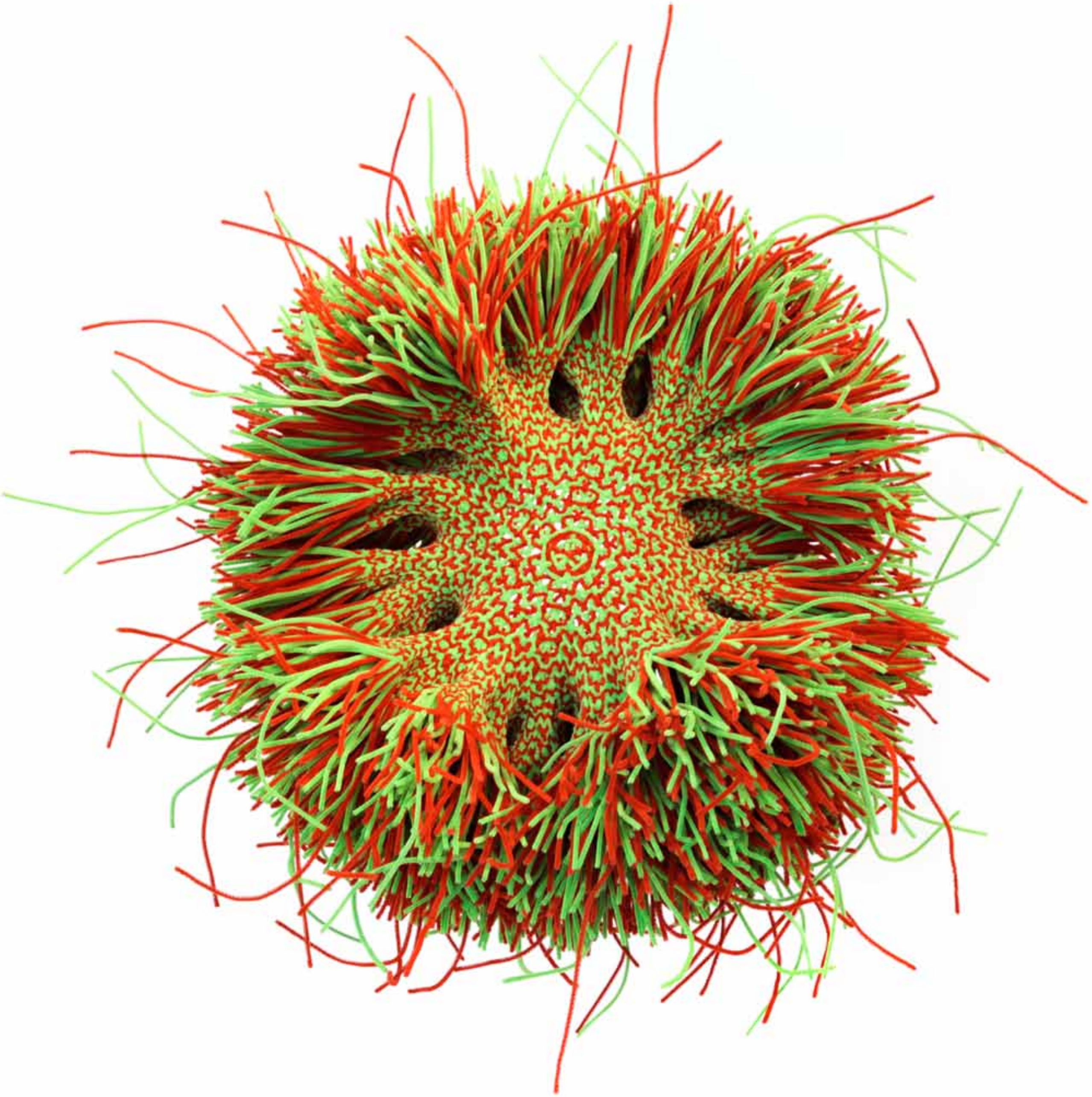
A PIPE CLEANER SCULPTURE: HYPERBOLIC STRUCTURE AND NEGATIVE CURVATURE

By the end of sophomore year at Cooper Union, Ryan and Trevor had become increasingly drawn to center-out elaborations: the way, for example, a low pressure weather system is densest in the center and progressively less so further and further out, while a high pressure system is the opposite. They decided to attempt a pipe cleaner representation of such a compounding system, denser and denser as it moved out from its center, always according to a consistent rule; and much to their surprise, following a few iterations, the structure began to fold in on itself, to ruffle, in hyperbolic fashion. It quickly became obvious, though, why this was happening. In their own words:

> The sculpture began with, at its center, a small 'seed' consisting of three green and three orange pipe cleaners crossed like spokes of a wagon wheel. Then, by inserting additional pipe cleaners and crossing and twisting them together, a fractal generating weaving algorithm thereupon kicked in simultaneously at six locations around the







wagon wheel, with the sculpture then growing outward in radiating concentric bands. In addition to crossing and twisting pipe cleaners, the algorithm required that additional pipe cleaners be periodically introduced such that their numbers double with each new circular band. The six original sites of the algorithm compounded into twelve sites in the second concentric band, 24 in the third band, and so forth. Eventually, these ever-doubling pipe cleaners pinched out the air space between themselves, sideswiping into each other in a massive project-wide collision, causing the surface to buckle in hyperbolic fashion into the third dimension simply in order to find more room.

The Twins would subsequently come to understand that the pipe cleaners were evincing a property mathematicians refer to as "negative curvature" (to distinguish things from its opposite, the sort of positive curvature one sees in a sphere). Nor was such curvature that unusual: indeed it pervades the natural world. As the mathematician Chaim Goodman-Strauss (University of Arkansas) wrote them, enthusiastically:

> I've been interested in just these sorts of growth rules for surfaces of negative curvature for some time, and it's wonderful to see such a beautiful example. In essence, this is precisely the way that real living surfaces (lettuce, scallops, coral, endless examples) of negative curvature grow — they can only increase the total amount of negative curvature through well-defined growth rules of this sort, along the fringe of the surface.

Furthermore, they'd learn that in mathematical terms they'd unknowingly ventured into a rich and vital field of inquiry cutting across computability theory, physics, complexity science, and theoretical biology known as "cellular automata." In classical terms, a cellular automaton is the simplest of computing machines, capable of changing its state from either *on* to *off*, or *off* to *on*, dependent on the states of neighboring automata according to a specific algorithmic rule. Many automata are arranged in a regular grid of *cells*, and the rule can be applied to the entire population simultaneously and repeatedly, thereby creating patterns that flow over the surface of the cells. Only, the Twins had added a twist. As the renowned physicist (and former Richard Feynman student and discoverer of the *real* quark) George Zweig wrote them:

> I think your pipe cleaner weavings are created with an algorithm like that used to animate cellular automata (CA), but with an important difference. The sites are fixed in a CA; only their values change. You add pipe cleaners to your weavings, which corresponds to inserting pairs of sites into CAs, as they evolve. If you want to be most helpful to mathematicians and programmers, I suggest that you write down the rules you use in creating your weavings. These rules should be in the form of algorithms, which means that either a human or a machine could create the weavings without further input. These algorithms would be the foundation for an extension of the theory of CAs to include site insertion and, of greater interest to you, the foundation for a software program that would allow you to create weavings in a virtual visual space. Then you could explore:
>
> - weavings woven forever, with decreasing "pipe cleaner" thickness;
> - the set of all possible algorithms that create weavings;
> - topological weavings that have intersecting surfaces;
> - God only knows what else.
>
> With the algorithms in hand, it should be relatively easy to get others involved.

In response to Zweig's request, the Twins generated an algorithmic recipe and accompanying diagrams, concluding that:

> Because all the activity occurs at the sculpture's outer rim, and because along the rim each pipe cleaner can be conceived of as a cell or point in a line, when translating the sculpture's assembly algorithm into a cellular automaton, only a one-dimensional cellular automaton is required: a one-dimensional line of cells, that is, that will connect back on itself in a ring, encompassing the sculpture's circular perimeter. Thereafter, with each new generation, the one-dimensional ring will move outward concentrically like rings of a tree... As with the physical sculpture, all previous generations will remain visible as progressively smaller concentric rings constituting the core of the cellular automaton.

A DESCRIPTION OF THE ALGORITHMIC RECIPE FOR THIS SCULPTURE

To begin with, a rule is established to limit the types of actions that can be performed: orange pipe cleaners will only be allowed to twist and join with other oranges, and greens with other greens. This means that two independent networks will come to comprise the sculpture, one orange, one green. The orange network and the green network will snake back and forth between each other, helping organize each other, and stabilizing each other, but never actually connecting.

With this rule in hand, an experimental weaving pattern can now be intuitively explored. Given that six identical sites surround the 'seed' at the core of the structure, a move (either a specific cross and twist, or a pipe cleaner addition) can be devised for one of the six sites and then simply repeated at all five of the others. This keeps the focus of one's attention very contained and local, and simplifies decision making while figuring out the pattern. Furthermore, all activity happens on the rim of the sculpture, so all attention is concentrated there. Specifically attention is paid to the order of alternating orange and green pipe cleaners as they dot along the rim, and to how that order changes as crosses and twists get made and new pipe cleaners added. For instance, at one moment the order arrived at will consist of:

2112112112112112112112112112112112112

and so on (which is to say, two orange pipe cleaners followed by one green, then one orange then two green, and so forth) until that order makes its way all around the perimeter and reconnects at the point where it started, forming a ring. At that particular moment, each double orange has a single green to both its sides, and vice versa: each double green has a single orange to both its sides.

Whenever that particular repeating order occurs, it is decreed that new pipe cleaners be added in both colors, following the rule that each add must first be folded in half to make a V. The crux of the V then has to be targeted between the doubled pipe cleaners of its color, whereupon the two sides of the V span left and right, skipping over the neighboring singles of the opposite color. Then, each end of the V gets twisted to join with the singles of its color one more neighbor down, which causes those singles to become doubles. Once such an add has been targeted at each initial double, all of the initial singles turn to doubles and the order becomes:

2222222222222222222222222222222222222

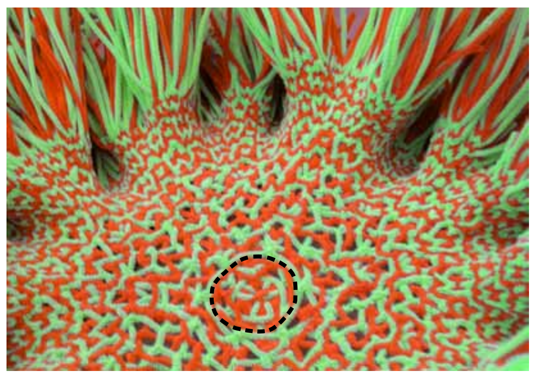
(which is to say, doubles repeating all the way around the new perimeter.)

Over the course of the sculpture other orders occur as well, each always with an internal repetition like the two mentioned above, with the repeating segment always bilaterally symmetric. Such repeating segments from other patterns include:

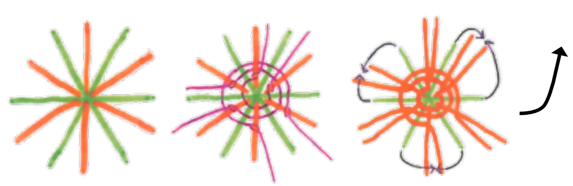
...2 1 1 2 2 2 1 1 2...

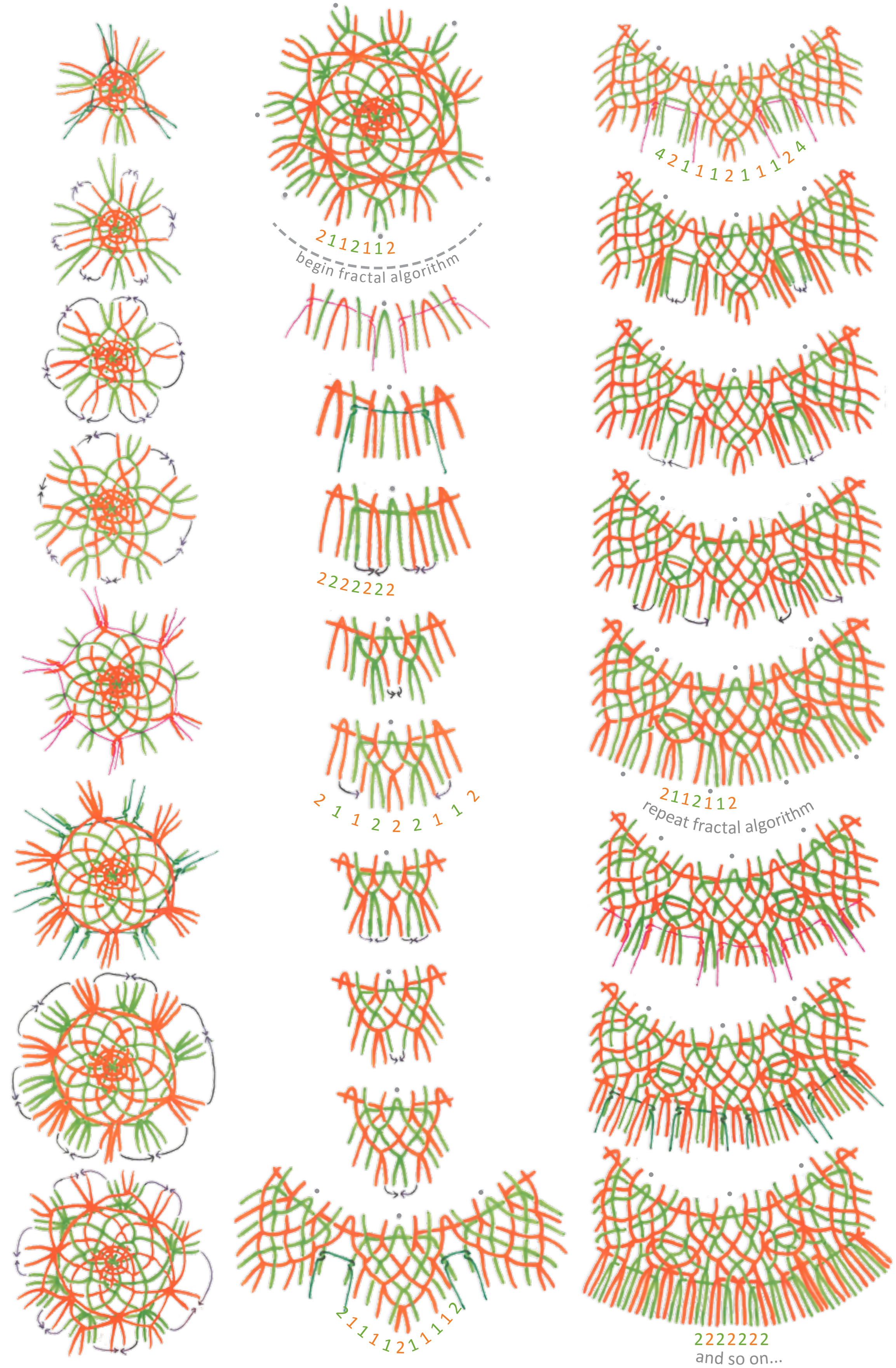
and ...2 1 1 1 1 2 1 1 1 1 2...

and even ...4 2 1 1 1 2 1 1 1 2 4...

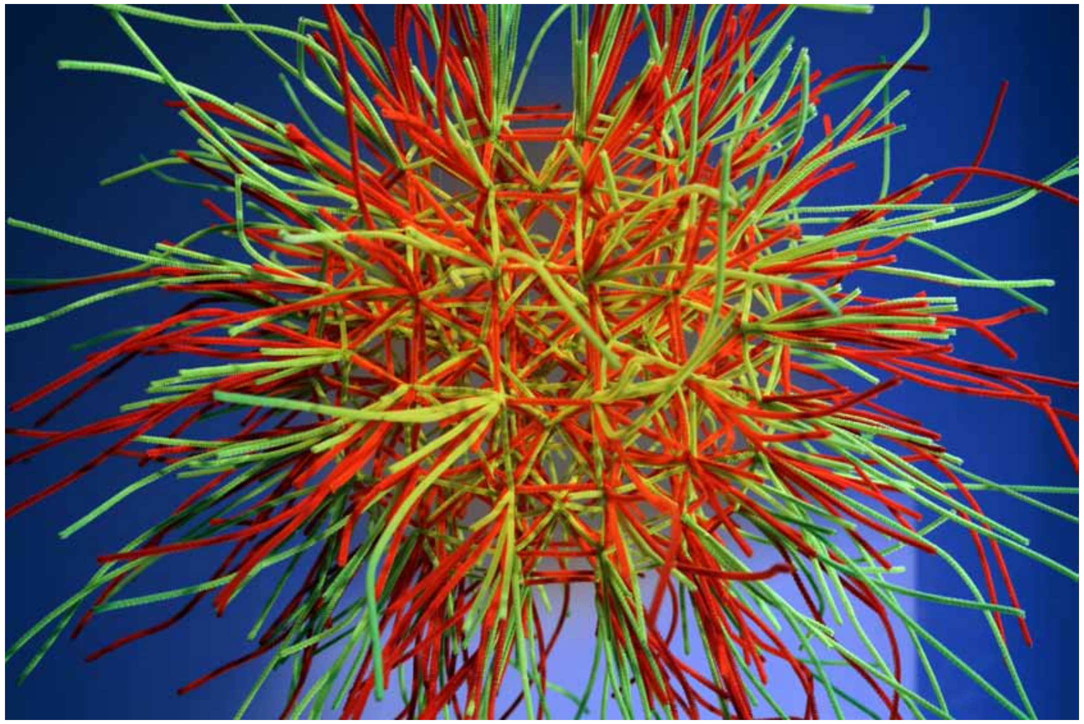


As can be seen in the diagrams that follow, the core, as it were, consists of twelve steps, at which point the structure reaches a threshold, indicated above by the dashed circle. At that stage, when the order has reached 2112112 at six sites around the perimeter (indicated by gray dots), a repeating fractal generating algorithm kicks in. Fourteen steps later the same 2112112 pattern recurs, this time at twelve sites around the perimeter. At each new site the algorithm then restarts.

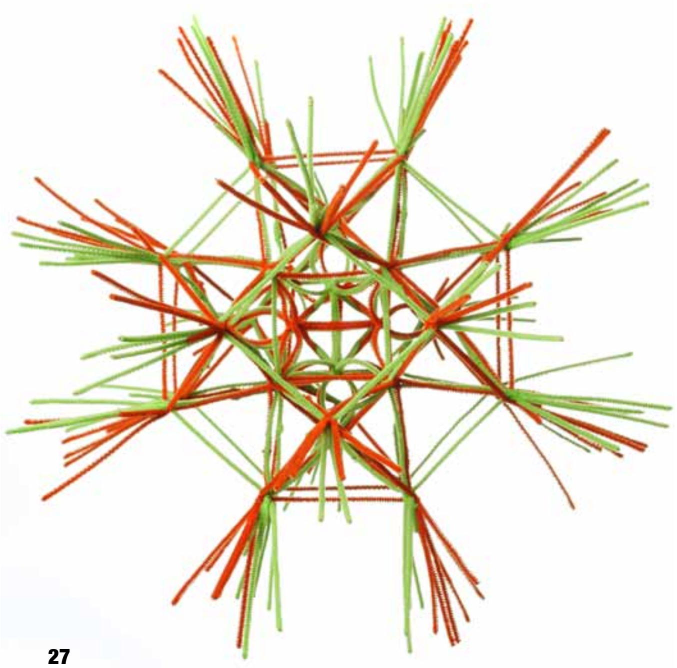




ANOTHER PIPE CLEANER SCULPTURE: VOLUMETRIC EXPANSION



While still at Cooper Union, the Twins conceived of another pipe cleaner sculpture, though one that they only got around to realizing in more recent years, one in which *volumetric* or *solid* (as opposed to flat) shapes (pyramids, cubes, and other such solids called *polyhedra*) might be seen to compound outward in an ever-expanding manner. In their words:

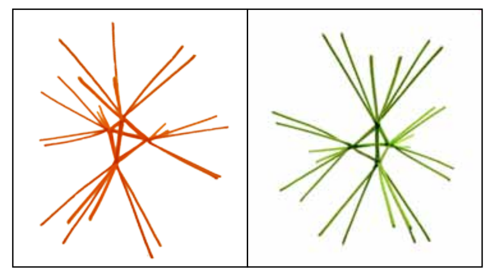


We were aiming for a version of the pipe cleaner sculpture that, instead of lying flat (at least initially) like the first one, would instead grow three-dimensionally into the full sphere of available space surrounding the 'seed' composing its core. At some gut level the seed that felt most solid volumetrically to us was the pyramid with a three-sided base, or *tetrahedron* as it's called. (It actually has *four* faces of course, a triangular base and then three triangular faces rising to a common vertex from each of that base's three edges). So this sculpture began with two tetrahedrons — one fashioned out of orange pipe cleaners, the other out of green — the two of them crisscrossed inside each other such that the vertices of one poked out of the faces of the other, like a three-dimensional Star of David. A cluster of six pipe cleaners sprouted from each vertex of both tetrahedrons, orange ones from the orange tetrahedron, and green from the other.

From there, developing a fresh algorithmic rule, the Twins contrived a compounding method whereby the structure kept growing outward, through a series of ever wider polyhedral volumes. As with the first sculpture, this cyclical pattern created a self-similar fractal structure, compounding by way of a network of pipe cleaners that smoothly multiplied as it grew outward, in this instance not into hyperbolic negative curvature, but rather into the opposite of negative curvature, which is to say, toward a spherical form.

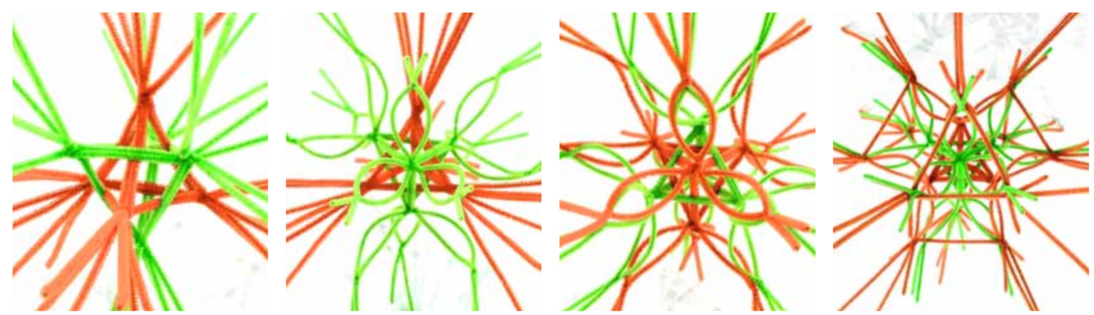
Interestingly, the Twins had hit upon an instance of a paradox famously associated with the great fifteenth century mathematical mystic Nicholas of Cusa. While the successively wider polyhedra that their algorithmic procedure kept generating with each outward iteration seemed to be closing in on the form of a sphere, in another sense, as Cusa never tired of pointing out, the more such objects expanded, the less like spheres they became. Such objects tend to multiply sides and angles exponentially (thereby becoming ever more complex), whereas a sphere by contrast is the very definition of simplicity, boasting only one smooth surface and no angles whatsoever. For Cusa this paradox comprised an allegory of the need for a "leap of faith" (his term, Kierkegaard got it from him), a leap outward, that is, from the ever more complicated polyhedron to the essentially simple sphere (which symbolized, in Cusa's formulation, oneness with God). Such a leap, Cusa in turn argued, could only be accomplished through grace. For Newton, a few hundred years later, these sorts of paradoxes would form the basis for the calculus, by way of which such compounding polyhedra did indeed arrive, at the limit of infinity, at the condition of a sphere, a fact which in turn would form the basis for some of the most powerful advances in the history of mathematical analysis.

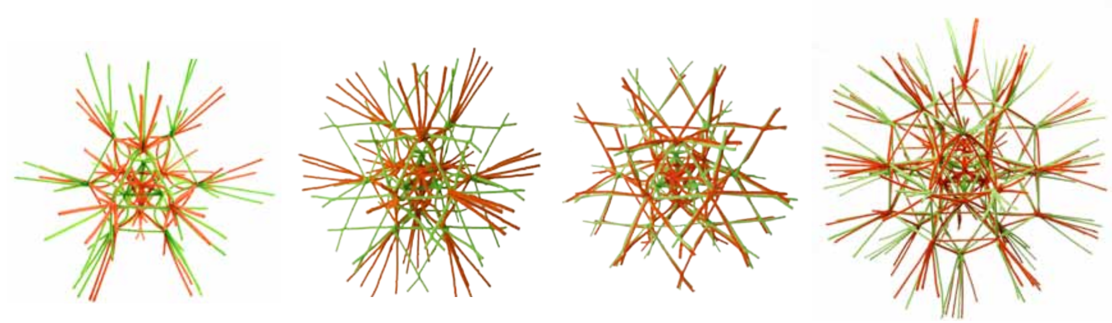
A DESCRIPTION OF THE ALGORITHMIC RECIPE FOR SCULPTURE TWO



So, as we say, this sculpture began with two tetrahedrons — one fashioned out of orange pipe cleaners, the other out of green — the two of them crisscrossed inside each other such that the vertices of one poked out the faces of the other, like a three-dimensional Star of David. A cluster of six pipe cleaners sprouted from each vertex of both tetrahedrons, orange ones from the orange tetrahedron, and green from the other.

The next step the Twins contrived involved uniting some orange and some green together in a new set of vertices one layer out. This was done by bending three flower-petal-like shapes out of the six pipe cleaners sprouting from each vertex, which caused them to disperse in pairs towards new vertices where it just so happened the tip of each orange flower petal neatly rubbed up against the tip of a green flower petal. Now, with two orange and two green pipe cleaners at each new vertex, additional pipe cleaners were added to connect the vertices. The





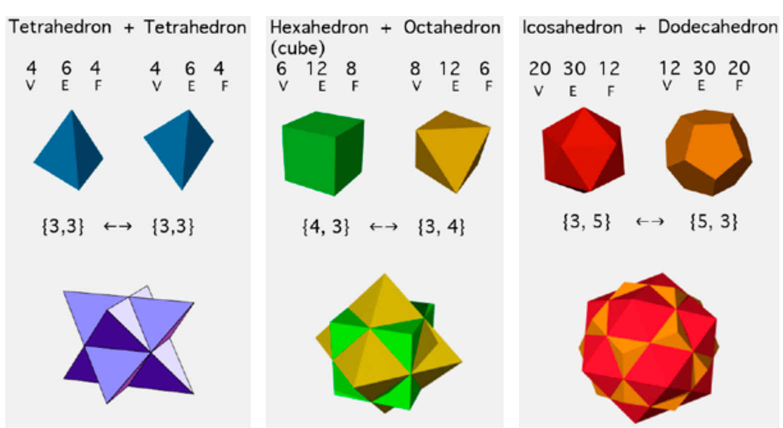
middle of each 'connector' pipe cleaner was planted in such a way that its two ends could be twisted one to each of the two vertices it connected, creating a cluster of four green and four orange at each vertex.

From this point forward, just as with the earlier ruffling pipe cleaner sculpture, a localized building rule, an algorithmic procedure that could be performed identically at every connector, was devised. The procedure is always only two steps long: 1. Join matching colored pipe cleaners from the two vertices separated by a connector into an equilateral triangle above the connector. This creates a new set of twice as many vertices one layer out, with two orange and two green at each vertex. 2: Add connectors in the manner specified above. This brings the arrangement back to where it started, with four green and four orange at each vertex, but now with twice the vertices and twice the connectors as the previous layer. Because the local arrangement at each vertex is exactly the same as before, the building procedure can now be repeated ––– this time at twice as many sites ––– which in turn will produce yet another set of double the vertices one layer further out. And so forth...

THE SECOND PIPE CLEANER SCULPTURE *(CONTINUED)*
COMPOUNDING POLYHEDRAL SOLIDS

The Twins recently spent a fascinating day with John Conway, the renowned Cambridge and Princeton mathematician (famous among other things for having radically invigorated the study of cellular automata with his invention of the digital *Game of Life*).

Observing this second of the Twin's pipe cleaner sculptures, Conway described for them a system that he had invented, or rather embellished, based on one originally devised by Johannes Kepler, for the classification of polyhedra. In this system two related polyhedra that could fit neatly into one another (because they both have the same number of edges, and the number of faces of one is equal to the number of vertices of the other)

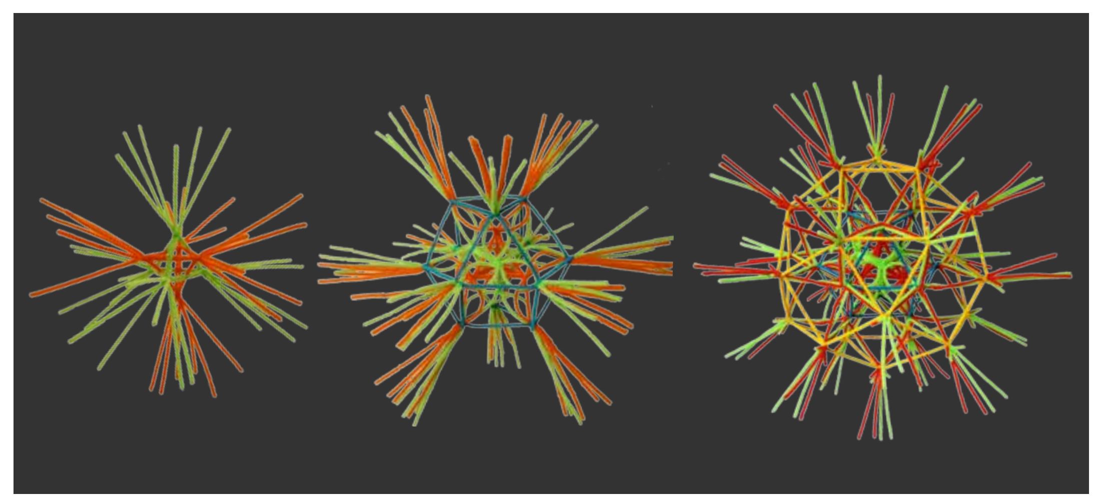


V = vertices, E = edges, F = faces. {X,Y} = vertex configuration

would be referred to as 'a married couple.' Then, a whole series of other polyhedra could be derived from the shapes created by that 'marriage,' solids which Conway in turn referred to as the couple's 'sons' and 'daughters.' A daughter is the polyhedron created when the points at the intersecting edges of the married couple are connected, creating a form *inside* the couple that is more complex than either of the married partners. A son is the polyhedron *outside* the couple that will perfectly wall in their two united volumes by creating faces out of the surfaces implied by their intersecting edges. (As Conway explains, "Female polyhedra are distinguished from male ones because they have more *faces* than vertices: F for Faces, F for Female. Male polyhedra, on the other hand, have more *vertices* than faces: V for Vertices, V for Virility.") The sons and daughters of the marriage may in turn marry other polyhedra they neatly fit with, thereby producing 'grandsons' and 'granddaughters'. Such lineages could be generated from a variety of root polyhedral marriages (a cube with an octahedron, for instance, or a dodecahedron with an icosahedron, or one tetrahedron with another, etc.), each of which would spawn a family tree of sons and daughters in successive generations that could then continue all the way till the end of time.

As it happened, the Twins' volumetric pipe cleaner sculpture turned out, entirely coincidentally, to manifest one of Conway's polyhedral family trees, in this case the one generated by the marriage of two tetra-hedrons, though through a method entirely their own. From this particular marriage of the orange and green tetrahedrons, Conway now went on to point out (literally, sticking his finger deep into the piece's burgeoning structure), first a notional son is born ("that cube in there implied by the vertices of the two tetrahedrons, see?"), which in turn becomes the father of the next generation, but from then on the progeny become a never-ending succession of daughters: daughter, granddaughter, great-granddaughter, and so on, one generation after the next. Each new daughter, he explained, appears as a new layer that fully encompasses the original marriage at the core of the sculpture as well as any previous generations of daughters, (which is unusual, for in his system daughters are always *inside* the married couple). No generations are skipped and each generation appears in order. Curiously, the husbands to the daughters are not physically manifest, but through an alternative method, the Twins' pipe cleaner algorithm establishes the same set of intersection points at which the wife's edges would meet those of her husband, and thus their daughter is still able to be geometrically derived in the sculpture's next layer outward. ("An Immaculate Conception," Conway crows, "how about that!")

Back home, for the purposes of this show, the Twins decided to make a new version of their second sculpture, one in which the founding marriage and subsequent compounding progeny would be successively represented by different colors, for easier viewing. The sequence of nested polyhedra emerge as follows:



Kepler/Conway polyhedra emerging amidst compounding pipecleaners

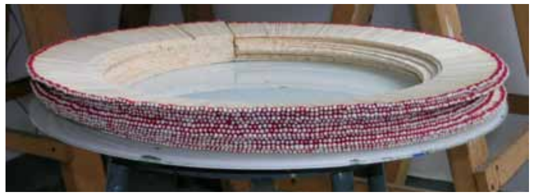
THE MATCHSTICK DOME

During junior year at Cooper Union, in another algorithmic exploration based on a simple procedural rule executed in repetition, Trevor and Ryan spent months fashioning an object out of roughly 9,000 matchsticks which, owing to the shape of the individual sticks (broader and round at the tip, narrower and square at the base) presently curved in on itself, initially as a circle over a table's surface, and eventually (as they added row upon row on top of that original circular row) like a dome. At some point in the process, the

Twins realized that every matchstick in the construction was aiming at a single imaginary focal point at the center of the hollow, like nothing so much, they further realized, as the way an infinite number of light rays shoot out from any given light source (be it the sun or a candle or a light bulb). For that matter, seen the other way around, the structure was reminiscent of the way an infinite number of light rays converge from all those sources into any given individual's eye. (For more on the implications of that discovery, see the discussion of Light Foam ahead.)

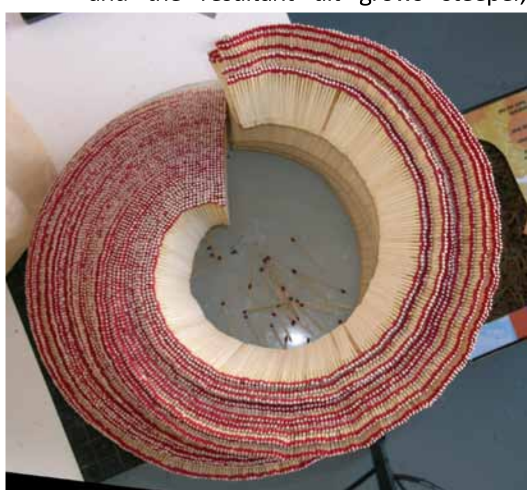
THE MATCHSTICK SPIRAL

Actually, the Twins had cheated a bit with the Matchstick Dome: at a certain point in its construction, they'd noticed that in fact the matches were beginning to veer away from the strictly domelike, that they seemed to want to do something else. For the Dome project, the Twins overrode that tendency by nudging them back into the desired form (by occasionally adding tiny gaps of air between the match heads). But a year later, curious, they returned to the project, constructing a second structure using the same rules but this



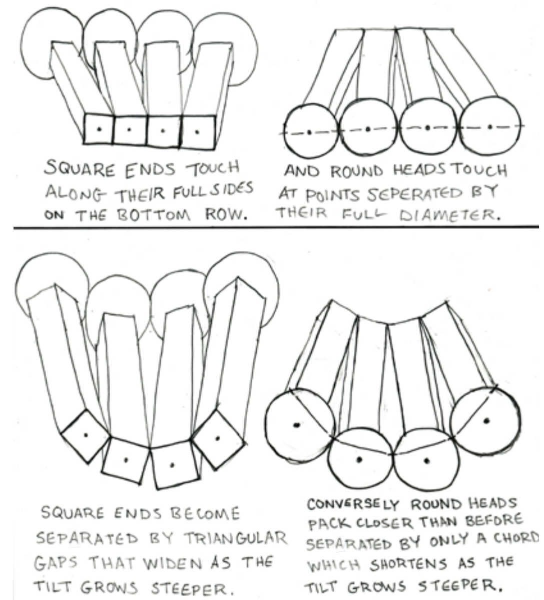
time allowing the matches, as it were, free rein to go any which way they wanted, and to their surprise, the wending rows began to express a *spiral*. Thinking about this, later on, Trevor realized:

> As horizontal row upon row gets added and the resultant tilt grows steeper,



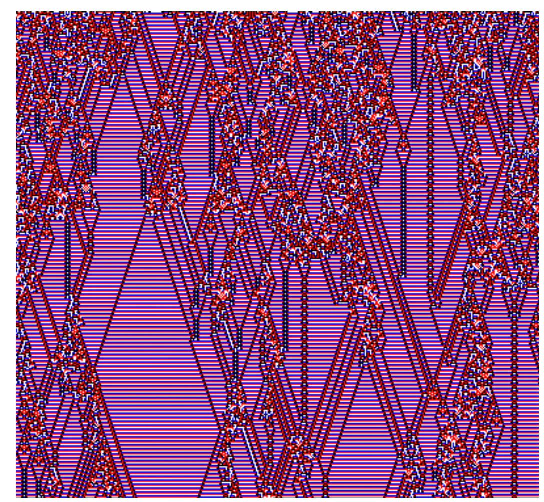
> it causes two opposing *gradations of packing density* to occur between the two ends of the match (*square* wooden stick and *round* match head). The round heads pack progressively closer under the progressive escalation of the tilt's pitch while the square sticks pack progressively further apart, which over time causes the emergence of the spiral form.

To elaborate this point, he drew a sketch:



But in fact, things were even stranger than that, for the successively compounding rows of matches weren't in fact resolving into a *simple* spiral either. Rather, somewhat mysteriously, they seemed to be jogging and lurching, getting ahead of or behind themselves as they rose from row to row, albeit with seemingly periodic regularity, although of a type that seemed totally unrelated to the rules generating their growth. To this day, the Twins aren't quite sure why this happens, though they do note the similarity of this seemingly randomizing

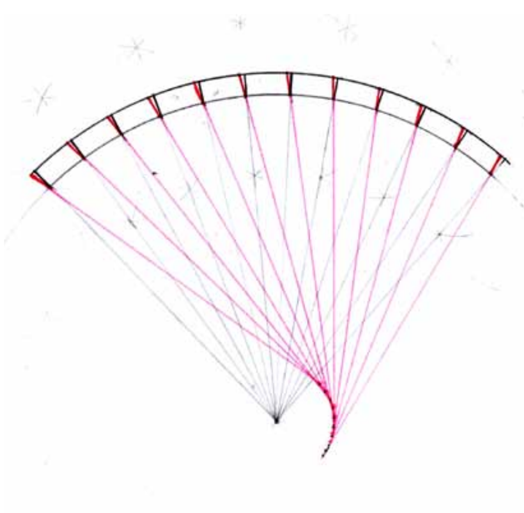
effect with the sort of thing theoretical physicist (and chief designer of the ubiquitous Mathematica software) Stephen Wolfram demonstrated in his 2002 book, *A New Kind of Science* (an exploration of the behavior of one-dimensional cellular automata not unlike John Conway's investigation of two-dimensional arrays of such automata in his *Game of Life*, described earlier). For his part, Wolfram describes the way in which certain sorts of cellular automata, while following simple regular algorithmic rules of propagation, start generating all kinds of chaotic and random-seeming outcomes down the line. See the following example from Wolfram's book:



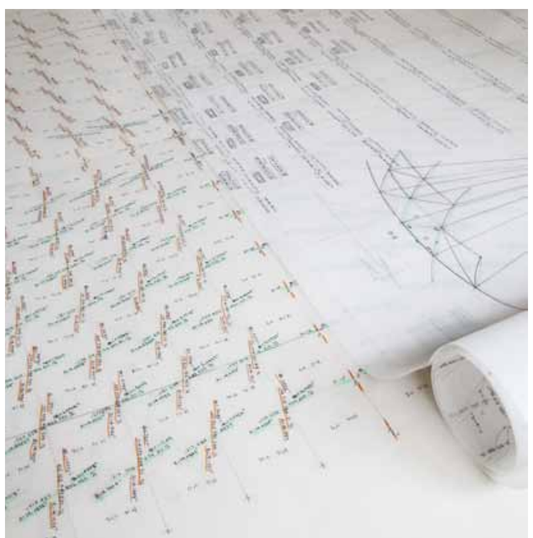
FASHIONING A MORE STURDY TRIPOD EASEL

Within a few years of leaving Cooper Union, the Twins came to recognize the need for a semi-spherical easel that would be both more sturdy (including a device for steadying the head while tracing the world before them), and more transparent to the viewing eye (with stiff sheet-metal slats to support the curved paper, pitched perpendicular to the sphere's surface, so they would foreshorten and appear as thin wires to an eye at the sphere's center, thereby blocking as little as possible of the scene beyond). Except it turned out that achieving transparency wasn't simply a matter of pitching all the slats perpendicularly toward the core. The problem didn't so much arise because of the way that the cells stretched and pinched from squares at the center of the grid into tighter and tighter diamonds as they wrapped to the grid's outer corners, though that presented significant challenges later on. Rather, the main complication, as the Twins discovered through exasperating trial and error, was that the slats had to be aimed *not* to the notional center of the sphere (as with the matchsticks or the corrugated hollows), even though that was going to be the place from where the viewer's *left* eye was to be steadily fixated on the intervening paper-filled cells throughout the drawing process; rather, the slats needed to be aimed at the viewer's *right* eye, which was going to be required to see *past* them to the world beyond. Slats aimed at the notional center would thus end up being seen at a glancing angle from the side and occluding slices of the world beyond, rendering a completely transparent gaze impossible from the point of view of the off-center right eye. To make matters worse, the right eye, unlike the left, was not going to remain steadily in place; over the course of the drawing, as the draftsman's head swiveled from right to left to capture the entire view, it would swing a subtly off-center arc about the on-center left eye. The slat angles therefore had to be recalibrated accordingly to aim toward the *moving target off-center eye*, with each successive slat essentially foreshortening toward a different point along that eye's arcing path.

There has to have been a simple formula whereby the Twins could have tabulated those steadily shifting required slat angles, but the Twins were not mathematicians, so, deploying a few spherical trigonometry equations they picked up from Steven Wolfram's mathworld.com, they ended up painstakingly calculating the proper pitch of each individual slat separately, one after the next, in an almost insanely time-consuming process, evidence of which can be seen in the charts below. (Anyone care to suggest how the boys could have contrived a simpler method?)



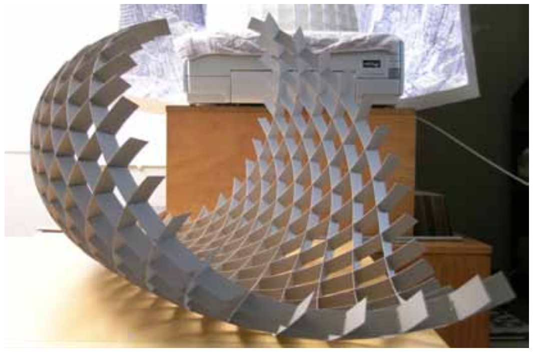
The right eye, unlike the left, was not going to remain in place over the course of the drawing



Sheets of painstaking calculations to determine the proper pitch of each slat aligned with the off-center eye

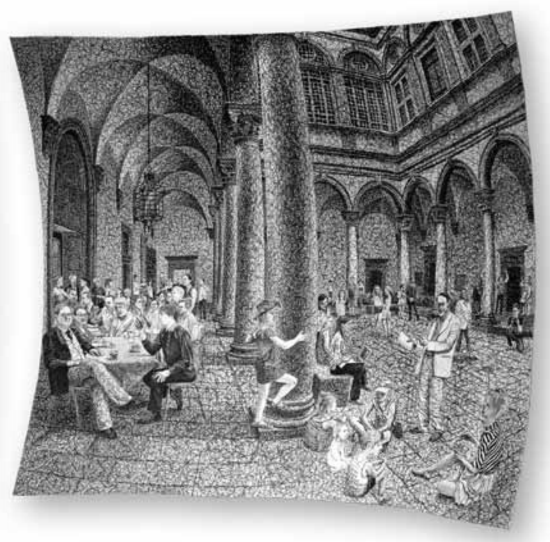
BOTCHED EASEL

Even with all their calculations, the Twins managed to bollix their first version of the metal grid. Although they'd tabulated the correct angles for each of the gradually slanting slats, when sketching those angles onto flat templates, they'd leaned half of them in the wrong direction, right instead of left — a situation they realized early on, but the resulting shape proved so beguiling in and of itself that they decided to go ahead and complete the mistaken version nonetheless.



THE PERFECTED EASEL AND ITS CONCAVE PROGENY

Once the Twins had perfected their gridded drawing easel (by 2005), they began taking it on the road—up to the top of the Chrysler Building, to a rooftop across the street from the Cooper Union's new building, out to Chicago and Los Angeles and beyond, and so forth—generating all manner of ever more ambitious curved drawings, each one taking from several weeks to over a month to complete. Initially, they limited themselves to contour drawings with blue or black pen, noting only the edges and sides of buildings, branches, plazas, interior spaces and the like, as with Anish Kapoor's Bean in Chicago's Millenium Park (pages 20 and 36); and the rendering of the great hall of Chicago's Field Museum (below left). The Field drawing includes myriad figures, from the museum's director to security guards and other friends met along the way, but note of course that they were not all in that one place at the same time: each posed separately across the many weeks of the piece's rendering. Gradually the Twins began introducing shading as well, as in their renditions of Robert Irwin's garden plaza at the Getty Museum in Los Angeles (page 3), and the central court of the Strozzi Palace in Florence, Italy (below right).



VANISHING POINTS ON THE INSIDES OF SPHERICAL SURFACES

Detail of Marina Towers drawing at right, from artist's original vantage

Have No Narrow Perspectives: Marina Towers

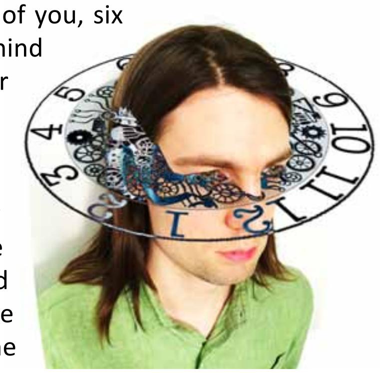
You may well have noted how straight lines in the world appear curved and pinched when projected onto the inner skins of these spherical surfaces. Although not exactly: if you lean into a curved drawing just right — which is to say get your face to exactly the place where the artist's was when he was drawing the piece — the curved lines will suddenly stiffen into precise perpendicular verticals and horizontals.

But this phenomenon in turn leads to an interesting side point about vanishing points: points to which parallel lines seemingly converge as they recede into the distance. This optical characteristic of visual space, which can also be illustrated by showing that objects seem to shrink in size as they approach the horizon, defines the root of many perspective systems developed during and before the Renaissance, and is an effect, the Twins posit, that is entirely caused by seeing via that *spherical* splay of light entering the eye. But then, how do vanishing points map onto *spherical* surfaces as opposed to the *flat* ones on which they were first deployed in the Renaissance? In their own words:

> The concept from linear perspective of 'vanishing points' to which parallel lines optically converge also exists when visual space is rendered onto a spherical picture surface. When drawn on the sphere, any straight line in the real world will become half of a *great circle* (which is to say a circle with its center in the center of the sphere) if it is extended to infinity in both directions. And, if you draw another line parallel to the first and extend it to both sides as well, it will intersect the first at 'infinity' to either side, which is to say, the two parallel lines will intersect at two polar opposite points 180 degrees apart on the sphere's surface. These two intersection points define the vanishing points not only for those two parallel lines but for all other lines in that 'family of parallels.'
>
> For instance, all vertical edges on a row of skyscrapers, if extended upward to infinity, will vanish to the 'north pole' of the spherical picture surface; conversely, their reflections into a smooth intervening pool would converge to the 'south pole' if extended downward to infinity. In other words, true vertical lines will always land on the spherical picture surface like lines of longitude on a globe, and when viewed by an eye at the globe's core, all of them will appear perfectly straight up and down.

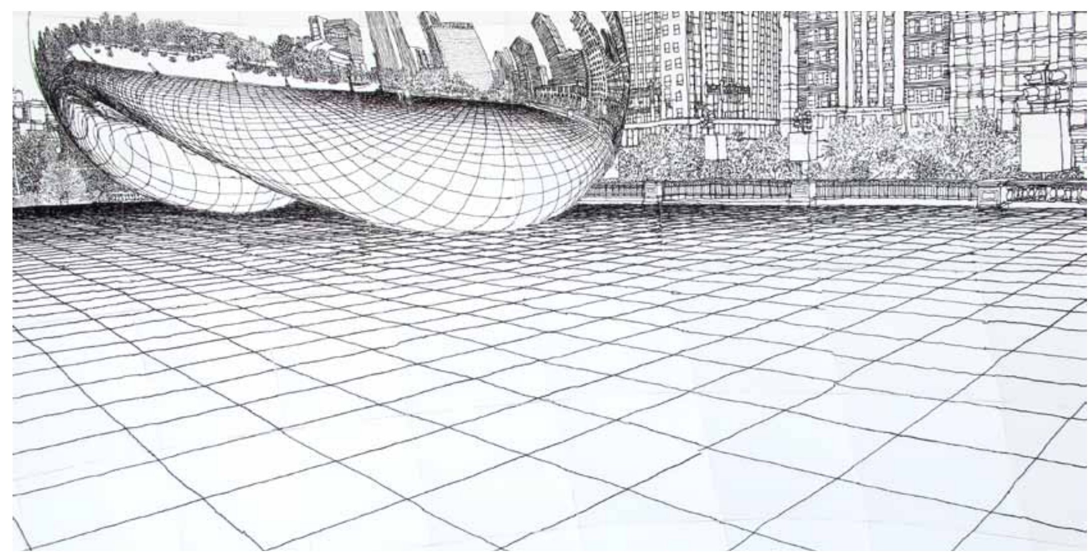
Another way you might picture this — again, standing with your eye at the center of the spherical expanse before you — is to imagine that you are bisected by a giant vertical clock slicing forward through your nose, such that the left side of your body is on one side of the clock-face and the right side of your body is on the other. The 'north pole' of the sphere centered on your eye would be at twelve o'clock above you and the 'south pole' at six o'clock below. The point is, you could rotate the clock face on its vertical axis and its forward edge would negotiate every vertical line in the world before you, every vertical vanishing up and down toward twelve and six: just like the so-called 'great circle' longitude lines on a globe (the circles being 'great' in that their center is the center of the globe).

Things become a bit more complicated, though, when thinking about horizontal lines. For one thing, they *don't* behave like latitude lines on a globe (which aside from the equator are not great circles, but rather smaller and smaller circles the farther north or south you go from the equator toward the single points at both poles). On a spherical picture surface, the horizontal lines will behave *just like* the great circles of the vertical lines, only precisely perpendicular to them. Therefore, now imagine a notional clock face horizontally slicing through your own head at the level of your eyes and ears: twelve o'clock is right in front of you, six is directly behind you, and your ears point to three and nine respectively.



Note that every horizontal line in the world (parallel to the surface of the earth, regardless of its height or direction) will converge on the rim of the clock face itself, which is to say on the horizon line. If you sight down a straight railroad track, its rails will converge at noon before you and at six behind you (as you can turn around and see). Now, if you stand perpendicular to the receding tracks in both directions, the tracks will converge at three and at nine, but still on the clock face, or rather the horizon line. Diagonal horizontal lines might converge at one o'clock (and seven) or two o'clock (and eight), but always on the horizon line.

Examining the Twins' drawing below of Kapoor's Bean in Chicago's Millennium Park is a good way to notice all of these effects.



THE COMING OF COLOR

Returning from a trip to California in 2010, the Twins informed friends that they had suddenly introduced color into their curved drawings, though the claim was initially somewhat undercut in that the main color they'd introduced in a pair of watercolor renderings of the midnight view out onto the Pacific from the Santa Monica shore, was black. "But black *as color*," they firmly corrected their interlocutors, "not as line." And indeed these images represented the world in the full density of its being, and no longer just as contour with maybe some added shading.

Presently the whole spectrum of the world's colors would come flooding into their purview. Occasionally, however, the Twins like reverting to a minimal vocabulary, as in another recent series of a stubbly fallow farm field in North Dakota, painted in the dead of winter. In this series, the palette is primarily white, in something of a nod to both Vincent Van Gogh and Anselm Kiefer.

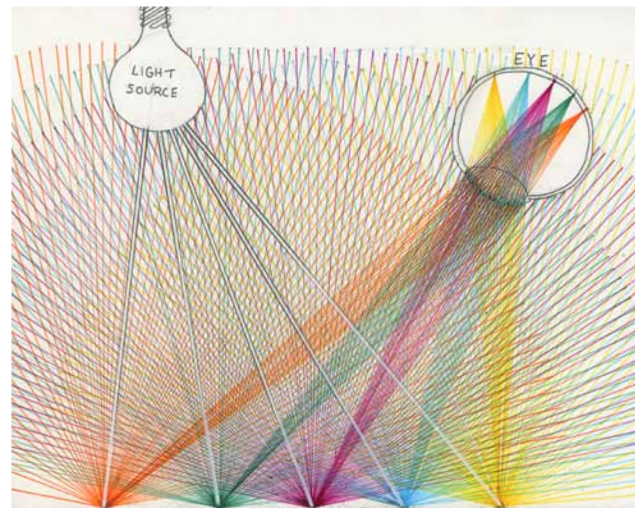
LIGHT FOAM

"The world is a dynamic mass of jiggling things, if you look at it right."
—Richard Feynman

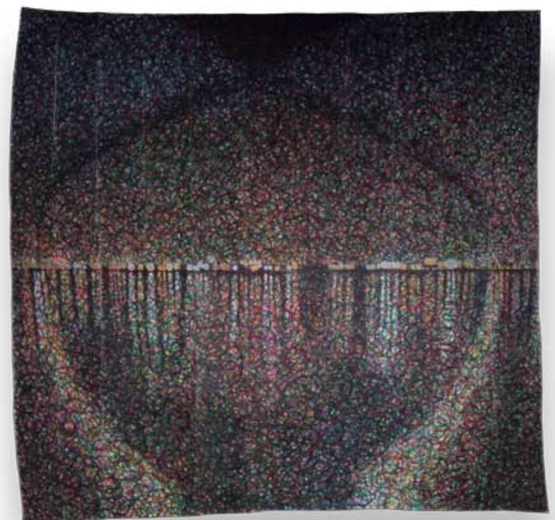
The more deeply the Twins investigated their new drawing technique, the more nuanced became their conception of the way light operates in the world. Because, as they came to understand:

> Light keeps exhibiting spherical behavior at multiple instances as it ricochets about the air. At the beginning of its journey (from the sun, say, or a candle), direct light bursts from its source in a sphere (of electromagnetic waves, to be precise). But upon hitting any single one of the gazillions of atoms composing surfaces in the surround, *a tiny sliver of that burst* of streaming photons scatters into another semispherical burst of ambient light, flung from the electrons of the atom at that point of impact. The many other neighboring atoms composing the illuminated surfaces each scatter their own semispherical ambient bursts, all of which overlap in the air, co-occupying the same air volume. When submerged into this pool of intermeshing semispheres of light waves, the lens of our eye simultaneously extracts a single lens-width slice from each of those gazillions of semispheres, and focuses it back to an individual point on the retina, thereby creating an image of all the atoms present in the eye's field of view: the familiar cone of vision.

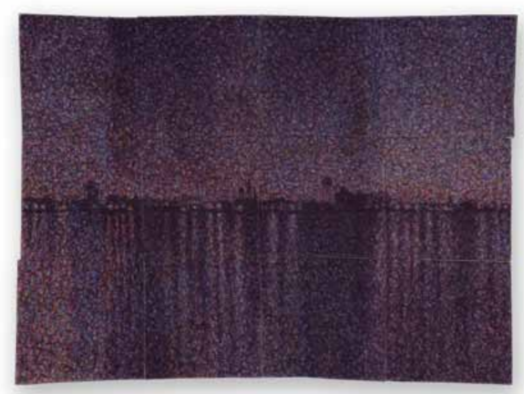
To better illustrate their point, the Twins diagrammed the overlapping semispheres of ambient light in the color sketch below:



Presently (already in some of their black ink drawings but ever more so with their efforts deploying colored ink pens), the twins began trying to evoke the wildly various "light foam" suffusing the atmosphere in the world before them by coiling threaded lines of overlapping colors into an ever denser mesh, often capturing the full spectrum of the world's colors by simply mixing ink from five or six such colored pens. The result reminded some people of Seurat's experiments with pointillism, to which the Twins responded, "Yes, except that Seurat painted that way at a time when people imagined atoms to be stacks of irreducibly tiny little marble-like pellets held in place, whereas we today are working at a time when such atoms are instead understood to consist in even more tiny subatomic nuclei surrounded by gaping voids of empty space, with light only showing us the electrons swirling at incredible speeds far on the outskirts of those otherwise mostly hollow atoms."



Brooklyn Nightscape II



Brooklyn Nightscape I

Brooklyn Nightscape I, detail

TWO OTHER COUNTIES HEARD FROM

As the years passed, the Twins would be informed of parallel instances of thinking by earlier visionaries from wildly different fields.

Thus, for example, one person turned them on to a remarkably apt passage from George Eliot's masterpiece *Middlemarch* (1874), and another to an uncannily resonant episode from a BBC documentary series about Richard Feynman in which the great physicist likens the transit of light rays moving every which way across the electrical field to that of water sloshing in a swimming pool as seen from the point of view of a tiny insect at the water's edge.

from GEORGE ELIOT'S *MIDDLEMARCH* {Chapter 24}

> An eminent philosopher among my friends, who can dignify even your ugly furniture by lifting it into the serene light of science, has shown me this pregnant little fact. Your pier-glass or extensive surface of polished steel made to be rubbed by a housemaid, will be minutely and multitudinously scratched in all directions; but place now against it a lighted candle as a center of illumination, and lo! the scratches will seem to arrange themselves in a fine series of concentric circles round that little sun. It is demonstrable that the scratches are going everywhere impartially, and it is only your candle which produces the flattering illusion of a concentric arrangement, its light falling with an exclusive optical selection. These things are a parable. The scratches are events, and the candle is the egoism of any person now absent.

{To which the Twins respond:}

A similar effect happens on rainy winter nights when a streetlamp is seen through a foreground tree. The veil of barren tree branches crisscrossing in all directions is drenched glossy wet by the rain. The light then reflects gleaming highlights within the thicket of dark branches in only concentric circles around the light! If you move your head, the concentric rings move with you.

Ceiling lamp reflected on a metal tabletop at Chipotle
Photo: Oakes

Streetlamp through a thicket of branches
Photo: Sekkle

RICHARD FEYNMAN ON LIGHT & VISION

{Episode 8 from Feynman's 1983 BBC series, "Fun to Imagine," produced by Christopher Sykes, slightly edited and condensed}

I'm sitting next to a swimming pool and somebody dives in… and before that, lots of other people have dived into the pool, so there's a very great choppiness of all these waves all over the water. And that gets me to wondering whether some sort of insect or something with sufficient cleverness sitting in a corner of the pool and just being disturbed by the waves, by the nature of the irregularities and bumping of the waves, could figure out who jumped in where and when and what's happening all over the pool.

Because that's what we're doing when we're looking at something. The light that comes out is waves, just like in the swimming pool, except in three dimensions instead of in the two dimensions of the pool surface. They're going in all directions. And we have an eighth of an inch black hole into which these things go, which is particularly sensitive to the parts of the waves that are coming in from a particular direction. The eye's not particularly sensitive when the waves are coming in at the wrong angle, which we say is from the corner of our eye. If we want to get more information from the corner of our eye, we swivel this ball about so that the hole moves from place to place.

It's quite wonderful that we figure everything out so easily. Granted, the waves in the water are a little bit more complicated. It would have been harder for the bug than for us. But it's the same idea: to figure out what the thing is that we're looking at, at a distance.

And it's kind of incredible, because when I'm looking at you, someone standing to my left could see somebody who's standing at my right. That is the light could be going right across this way—the waves are going every which way—right left up down perpendicular and so forth—it's just a complete network. Now it's easy to think of them as arrows passing each other, but that's not the way it is, because all it is actually is this entire field that's vibrating—it's called the electric field but we don't have to bother with what it is— it's just like the water height, going up and down—so there's some quantity shaking about in a combination of motions that's unbelievably elaborate and complicated and yet whose net result is to produce an influence which makes me see you, at the same time completely undisturbed by the other influences that are allowing this other guy over to my left to see the one to my right.

So that there's this *tremendous* mess of waves all over in space: all the light bouncing around the room and going from one thing to the other. Because of course most of the room doesn't have eighth-inch black holes. It's not interested in the light, but the light is there anyway: it bounces off this and it bounces off that, and all this is going on, and yet we can sort it all out with these little eighth-of-an-inch-hole instruments in the middle of our faces.

But beside all that, it turns out that the eye is only using waves between this length and that length, around a hundred-thousandths of an inch. What about the waves that go more slowly, that have a longer distance from crest to trough? The shorter waves are blue, and the longer waves are red— but when it gets longer than that we call it infrared— that's the heat. We feel those waves but our eye doesn't see them. Those pit vipers they got down here in the desert, they have a little thing that they can see the longer waves and pick out mice by their body heat, by looking at them with this "eye," which is the pit of the pit viper. But we can't—we're not able to do that.

And then these waves get longer and longer, and they're all through the same space, all these things are going on at the same time. So that in this space there's not only my vision of you, but information from Moscow Radio that's being broadcast at the present moment and the singing of somebody from Peru. All the radio waves are just the same kind of waves, only longer waves.

And there's the radar from the aeroplane flying up there above which is looking at the ground to figure out where it is, which is coming through this room at the same time. Plus the x-rays and cosmic rays and all these other things—which are the same kind of waves, exactly the same waves, but shorter, faster or longer, slower. So this electric field, this vibration, contains this tremendous information. And *it's all really there* —that's what gets you.

If you don't *believe* it, then you pick a piece of wire and connect it to a box. And in the wire the electrons will be pushed back and forth in this electric field, and you turn some knobs on the box to get the sloshing just right. And you hear Radio Moscow.

And you know that it was there—how else did it get there? It was there all the time. It's only when you turn on the radio that you notice it.

All these things are going through the room at the same time, which everybody *knows*. But you've got to stop and *think* about it to really get the *pleasure* about the complexity—the *inconceivable* nature of nature.

LIGHT FOAM COLOR CONCAVE DRAWINGS

The swirling line and compounding colors in many of the twins' recent drawings (often based on the output of no more than five or six differently colored pens) were their way of trying to evoke the light foam roiling just before our eyes, a phenomenon regarding the reality of which we are largely unaware because the atmosphere itself appears completely clear. As they point out, "Light passing before your eyes is invisible. It is only those rays that spear directly into your pupils (at literally the speed of light) that become seeable." Furthermore, in this series of recent color sketches, the Twins have tried to illuminate a very simple — yet overlooked — truth about the invisible light foam:

> We simply decided to show the light coming off every object in the drawing, also present simultaneously (in its color but less dense) at every other point in the air, filling the rest of the drawing. This inevitably leads to many, many coiling threaded lines of color overlapping to compose themselves into a dense mesh, which can be a scary amount of visual information to have to deal with. But in these sketches we tried it out, and it's been producing some interesting effects, in particular all sorts of curious melding colors (for instance, a crazy pink orange and a wonderful pale gray green, both of which we love), but also more generally, a look that is incredibly ethereal, eerie, dreamlike, but that really makes you pay attention.

Central Park, detail

In terms of the light foam diagram on page 38, it's as though the swirling lines exist as a cross section of the light *just outside the lens of the eye*, where the many beams are all still mixed together, before the lens separates them, focusing each individual beam onto an individual point on the retina.

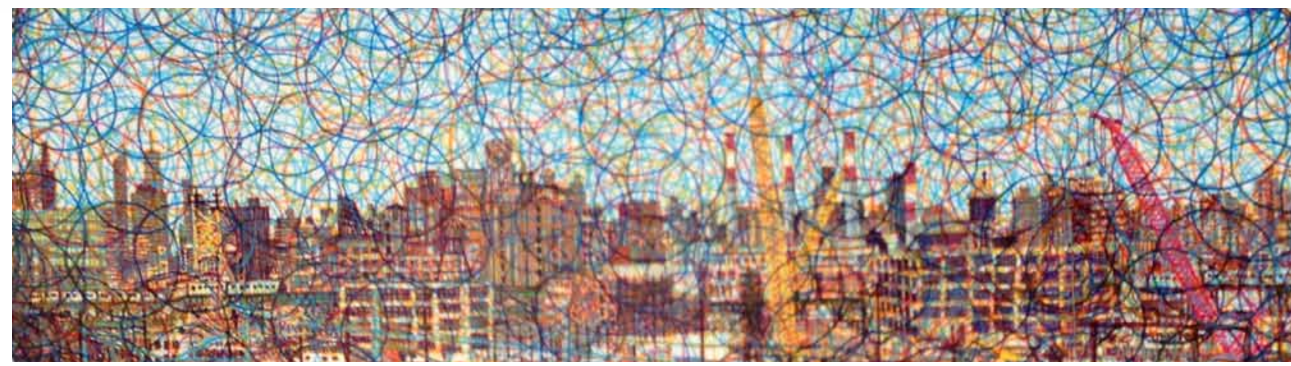
Across several early sketches deploying this method, the Twins also experimented by evoking the constrained part of one's field of vision with which we experience full depth perception, a shield-like space bordered as it is by the barometric pressure of one's occluded nose to either side. (See page 11 in the VQR piece).

More recently, in one of their most ambitious drawings, which they were working on over several months this past year, the Twins endeavored to capture the sense of time itself passing in their chronicle of the passing seasons playing across Central Park as seen from a west side balcony, from lush green late summer on the right side of the drawing, across vividly various autumn, to stark bare-limbed winter on the left side – to be filled in next year, since busy with other projects, they missed this year's winter iteration.

You may have noticed how this latest of their drawings lacks the light foam swirls of the immediately prior efforts. What's up with that? Well, as the Twins will tell you,

> The truth is that to create real light foam you don't actually have to draw it. Drawing it is in fact redundant because every dot made with a pen already evinces it. Looked at closely, this drawing is made up of thousands upon thousands of minute dots. But every one of them is more than just a dot. Each one instigates a dome of light foam that lifts off the paper and fills the entire room. No matter where you stand you can see each dot (though you may need binoculars if you're across the room). Consequentially even this drawing, and for that matter any drawing ever made, ramifies into a sculptured suffusion of light foam, whether that was the artist's intention or not.

Opposite, counterclockwise from top right: Skillman Avenue Studio View, Skillman Avenue Studio View in Winter, detail, Central Park (in process)



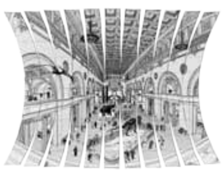
THE HYPERBOLIC SADDLE

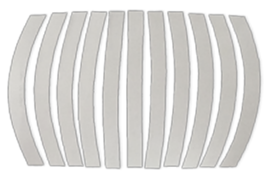
Here's another interesting thing Ryan noticed one day while playing with the vertical strips of one of their drawings, detached from their concave armature and laid flat one beside the next on a table. The seams between the flattened vertical strips, thus separated, evince widening v-shaped gaps, pinched at the middles and spreading toward the tops and bottoms. Once rejoined along their seams, of course, the gaps will close and the strips curl back into the tranche of a sphere. Turn the entire set of strips upside down as a group, such that the tops of the strips are now on the bottom and vice versa, and they will still form the tranche of a sphere when tightened up, only with the image upside down. However, if each flat strip is first lifted, *flipped face down*, and set back into the same place, such that each has been rotated about an individual vertical axis within itself (the way a set of vertical window blinds might rotate), the resulting sequence, lying flat, will bow out barrel-shaped instead of pinching in the middles like a butterfly (the empty gaps now swelling across the middles and pinching at the tops and bottoms). And if the seams are now tightened up in that formation, the result, instead of the tranche of a sphere, will be a *hyperbolic saddle*, a shape the Twins like to characterize as similar to that of a Pringles chip (as evinced by their wry couplet celebrating this phenomenon: "From the sphere to the hyperbolic Pringles chip, in one flip!"). Even John Conway was perplexed by this result.

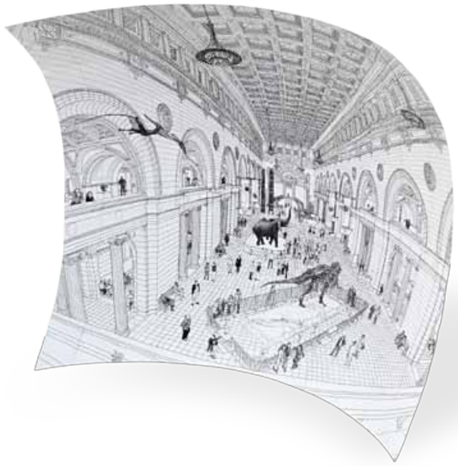
Curiously, this phenomenon does *not* occur if the vertical strips in question are the more commonly used *longitudinal sections* one sees on beach balls and lemon wedges, that pinch *to a point* at both the North and South Poles of the sphere. Imagine a series of such lemon wedge peels from half of a sphere uncurled to lie flat: flip those face down around their vertical axes and tighten them up, and you will still get a hemisphere. However, the perimeter shapes on the Twin's strips were instead produced by a choice they made in order to link their drawing surface conceptually with the traditional flat rectangular picture plane. Seeking to be just one step removed from that deeply canonized rectangle, they had simply taken a regular flat grid of two-inch square cells and *wrapped it* onto a sphere, a maneuver that sent the corners pinching into diamonds. Their vertical strips are thus columns of that once-square grid, with each strip outward, proceeding left and right from the still-vertical middle column bowing slightly more than the last.

So what is it about these progressively bowing strips of theirs, the Twins have recently taken to wondering, that, once flipped, has them tightening into the sphere's *obverse*, a Pringles saddle? One relevant difference may be how the borders of the bowed strips curve over the sphere's surface; that is, if one walked along a strip's edge, one's path would consistently bend to the left or to the right across the sphere, while the longitudinal sections' borders consist of straight paths, also known as great circles.

The Twins welcome your ideas on this subject. What's particularly intriguing to them is the way that spherical space and hyperbolic space seem to be connected by a 'grid logic' that originates on a flat plane. Mathematicians categorize spaces into three separate types: flat, spherical, and hyperbolic. So to discover that sectioning the sphere with a logic from the flat grid allows its surface to transform directly into a hyperbolic saddle, thereby involving all three types of space — well, as the Twins say, "That's pretty cool!"







THE FLATIRON BUILDING: A NEW DRAWING PROJECT

Just outside the Museum's front door and diagonally southwest across the park, you come to the very spot where Edward Steichen framed his hauntingly iconic 1904 photograph of horse drawn carriages languoring in the cool damp of an early winter evening, silhouetted in front of the Flatiron Building (at that time only two years old), below left. But be careful: these days you're likely to run into the Twins themselves out there, for they have chosen that very spot from which to frame their own concave tribute to the Steichen vantage, below right. They will be there periodically most days of this show, drawing away at their tripod easel. And as they complete each new cell, we will photograph it, blow up the photo, and attach it to an empty grid back in the gallery, so we can all keep track of developments.

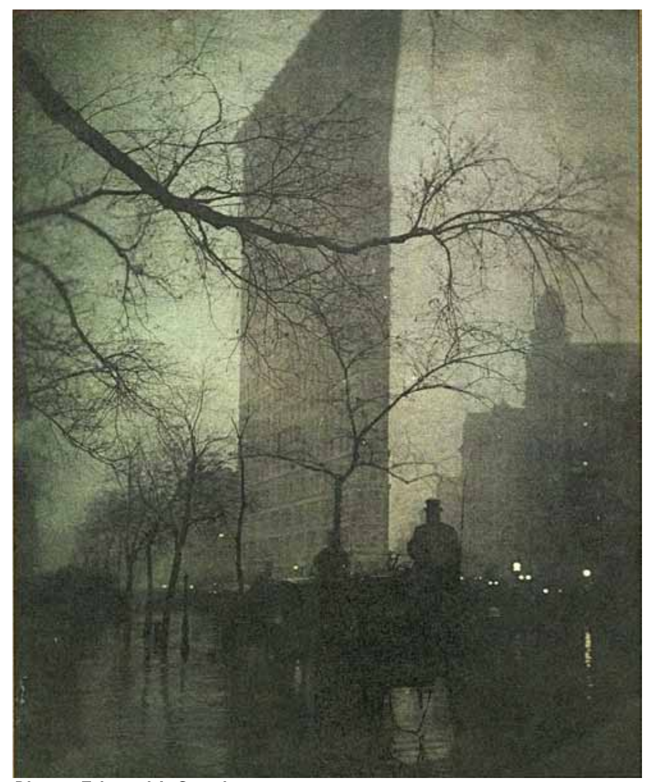


Photo: Edward J. Steichen

Photo: Andre Gauthier

Left to right: Trevor Oakes, Ryan Oakes, and Lawrence Weschler

For more information on the twins or Mr. Weschler, see their respective websites at oakesoakes.com and lawrenceweschler.com